

2008

MATHEMATICS (Speciality)

(For Commerce)

Full Marks : 70

Time : 3 hours

The figures in the margin indicate full marks for the questions

Parts of the same question should be answered together

1. (a) (i) Find the gradient of the line
 $3y - 5x + 7 = 0$ 1
- (ii) In a GP series the first term is 7, the common ratio is 2 and the last term is 448. Find the number of terms. 2
- (b) If $m = a^x$, $n = a^y$ and $a^2 = (m^y n^x)^z$, show that $xyz = 1$. 3
- (c) Prove that the points $(-2, 5)$, $(5, -2)$ and $(10, 10)$ are the vertices of an isosceles triangle. 3

(d) If $x = 2 + 2^{2/3} + 2^{1/3}$, show that

0

$$x^3 - 6x^2 + 6x - 2 = 0$$

5

Or

(e) Prove that the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, when

$$\frac{1}{a} + \frac{1}{b} = 1$$

4

(f) Derive an equation of a straight line passing through two given points.

6

(g) If a straight line makes an angle 45° with the X axis and if it passes through $(2, -1)$, find the equation of the straight line.

4

2. (a) (i) If

$$f(x) = \frac{x^2 - 9}{x - 3}$$

is continuous at $x = 3$; what should be the value of $f(3)$?

1

(ii) Find the value of

$$\int \frac{1}{\sqrt{2ax}} dx$$

2

(iii) Find $\frac{dy}{dx}$, when $ax^2 + 2hxy + by^2 = 1$

3

(iv) Evaluate : 3

$$\int \sqrt{5x+7} dx$$

(b) Find the maximum and minimum values of the function

$$2x^3 - 21x^2 + 36x - 20 \quad 5$$

Or

(c) (i) If $y = e^{mx}$, find $\frac{dy}{dx}$. 1

(ii) Find the value of

$$\lim_{x \rightarrow \infty} \frac{5 - 2x^2}{3x + 5x^2} \quad 2$$

(iii) Evaluate : 3

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$$

(iv) If $y = Ae^{mx} + Be^{-mx}$, show that

$$\frac{d^2y}{dx^2} - m^2y = 0 \quad 3$$

(d) Evaluate : 2+3=5

(i) $\int \frac{dx}{a+bx}$

(ii) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

where $u = x^2y + y^2z + z^2x$

3. (a) (i) Find the value of

$$\begin{vmatrix} 2 & 4 & 6 \\ 3 & 6 & -5 \\ 5 & 10 & 1 \end{vmatrix}$$

1

(ii) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = xy$$

2

(iii) Find $2A - 3B$, where A and B are matrices

$$A = \begin{bmatrix} 5 & 2 \\ 7 & -9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & -2 \\ -3 & 5 \end{bmatrix}$$

3

(iv) Solve by Cramer's rule :

3

$$2x + 3y = 13$$

$$x + 7y = 23$$

(b) If

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

show that $A^2 - 4A - 5I = 0$.

5

Or

(c) (i) When two matrices are said to be conformable for multiplication?

1

(ii) Find the value of

$$\begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 3 \\ 5 & -7 & 2 \end{bmatrix} \quad 2$$

(iii) Solve : 3

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = 0$$

(iv) Show that the matrix A satisfies the matrix equation

$$A^2 - 5A + I = 0 \quad 3$$

(d) Find A^{-1} , when

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix} \quad 5$$

4. (a) (i) Define a feasible solution and an optimal solution. $1\frac{1}{2} + 1\frac{1}{2} = 3$

(ii) Discuss the uses of LPP in business and commerce. 5

(b) Solve graphically : 6

Maximize $Z = 10x_1 + 5x_2$
subject to

$$2x_1 + x_2 \leq 26$$

$$2x_1 + 4x_2 \leq 56$$

$$x_1 - x_2 \leq -5$$

$$x_1, x_2 \geq 0$$

Or George Dantzig

- (c) (i) Who first developed LPP? Under what assumptions LPP may be solved? 1+4=5
- (ii) Discuss the limitations of Linear programming. 4
- (d) A company manufactures two types of products A and B. The manufacturing time required, the profit and capacity available at each work centre, are given below :

Work centre → Product ↓	Matching	Fabri- cations	Assembly	Profit per unit (in Rs)
A	1 hour.	5 hours	3 hours	80
B	2 hours	4 hours	1 hour	100
Total capacity	700 hours	1800 hours	900 hours	

Formulate the LPP to maximize profit. 5

5. (a) (i) If $\log 4 \cdot 596 = 0 \cdot 6586$, what will be $\log 45 \cdot 96$? 1
- (ii) If $\log 2 = 0 \cdot 30103$ and $\log 3 = 0 \cdot 47712$, find $\log 72$. 2
- (iii) If $a^2 + b^2 = 18ab$, show that

$$\log \frac{a-b}{4} = \frac{1}{2}(\log a + \log b) \quad 3$$

- (iv) To endow a scholarship of Rs 4,500 per year, what sum should be set aside in a bank reckoning 9% per annum rate of compound interest? 3
- (b) A housing flat costs Rs 1,36,000, 40% to be paid at the time of purchase and the balance reckoning CI @ 9% p.a. is to be paid in 12 equal annual instalments. Find the amount of each instalment. [Given $(1.09)^{-12} = 0.3558$]. 5

Or

- (c) (i) Fill up the blank : 1
$$\log_{50} 1 = \frac{0}{\quad}$$
- (ii) If $\log_a 8 - \log_a 6 + \log_a 9 = \log_a N$, find N . 2
- (iii) A person borrowed Rs 50,000 at 5% simple interest and invested the whole amount at 5% compound interest. What will be his profit in 2 years' time? 3
- (iv) Define perpetuity and deferred annuity. $1\frac{1}{2} \times 2 = 3$
- (d) A person borrowed Rs 20,000 at 4% compound interest and agreed to repay both the sum and the interest in 10 equal annual instalments at the end of each year. Find the amount of each instalment. 5
